# *16-350*
# *Planning Techniques for Robotics*

# *Planning under Uncertainty: Expected Value Formulation*

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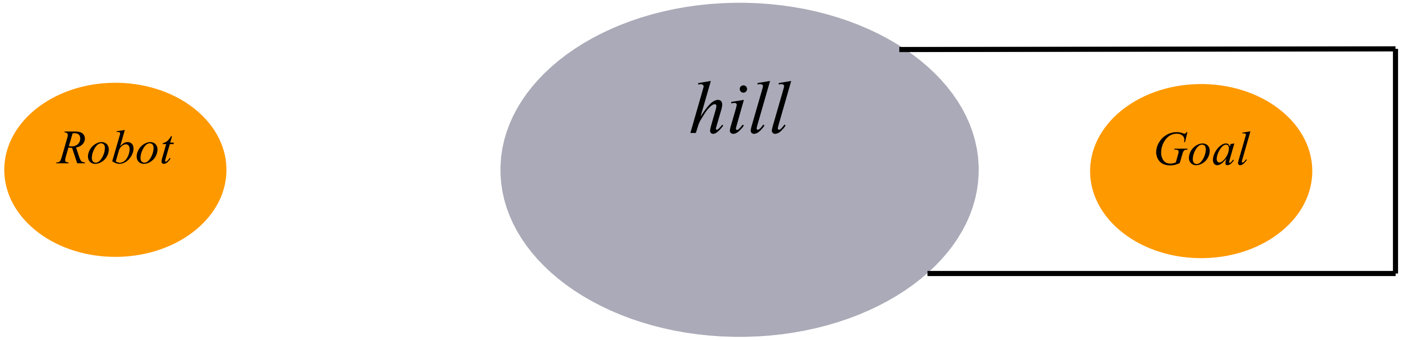
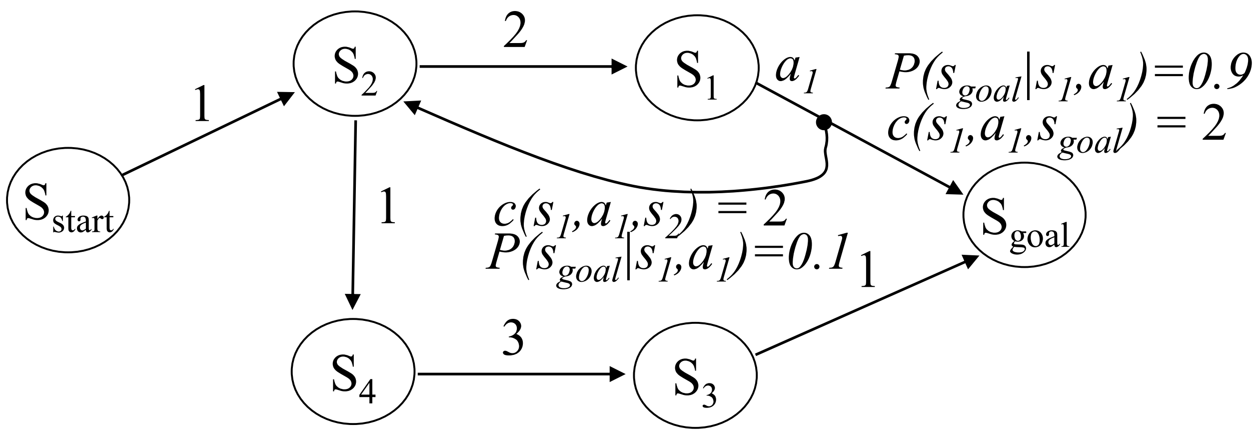
# Minimax Formulation is Often Too Conservative

*Example:*

*moving over the hill has 10% chance of slipping*



# Expected Cost Formulation




- Optimal policy $\pi^*$:
  minimizes the *expected* cost-to-goal
  $\pi^* = argmin_\pi\, E\{cost\text{-}to\text{-}goal\}$

*expectation over outcomes*

# Expectation of a Random Variable

*X - a random variable*
*E{X} - expected value of X (e.g., if you were to draw infinitely many samples of X, what would be the average of the drawn values)*

$E\{X\} = \Sigma_i X_i P(X_i)$ ($X_i$ *- all possible values of X*)

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- Suppose you roll a die. What is the expected value of the die?

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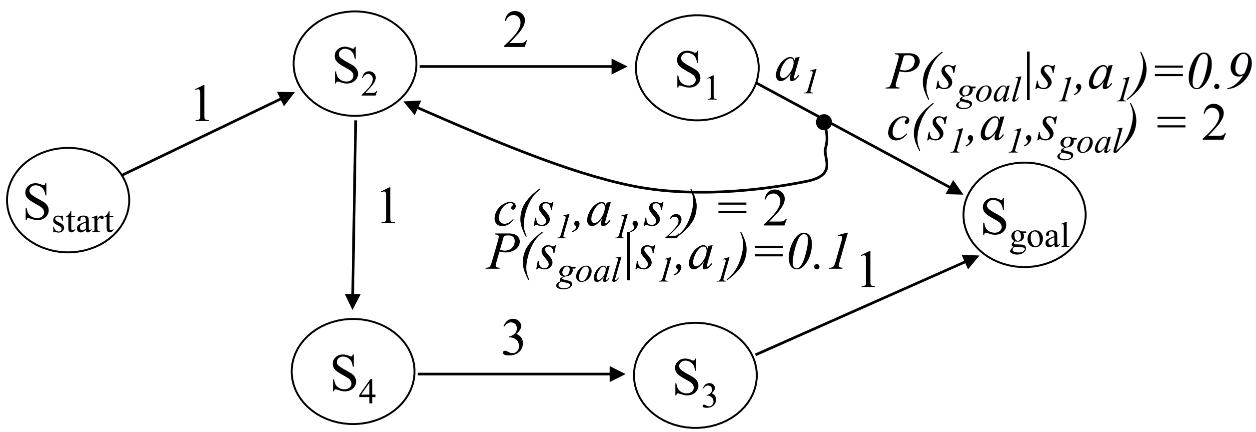
- Suppose you get a <u>random</u> coin from a pool of 5 nickels, 3 dimes and 2 quarters? What is the expected amount you get?

# Expected Cost Formulation

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*expectation over outcomes*

- expected cost-to-goal for $\pi_1$=(go through $s_4$) is

  1+1+3+1=6

- cost-to-goal for $\pi_2$=(try to go through $s_1$) is:

$0.9*(1+2+2) + 0.9*0.1*(1+2+2+2+2) + 0.9*0.1*0.1*(1+2+2+2+2+2+2) + \ldots = 5.44\overline{4}$

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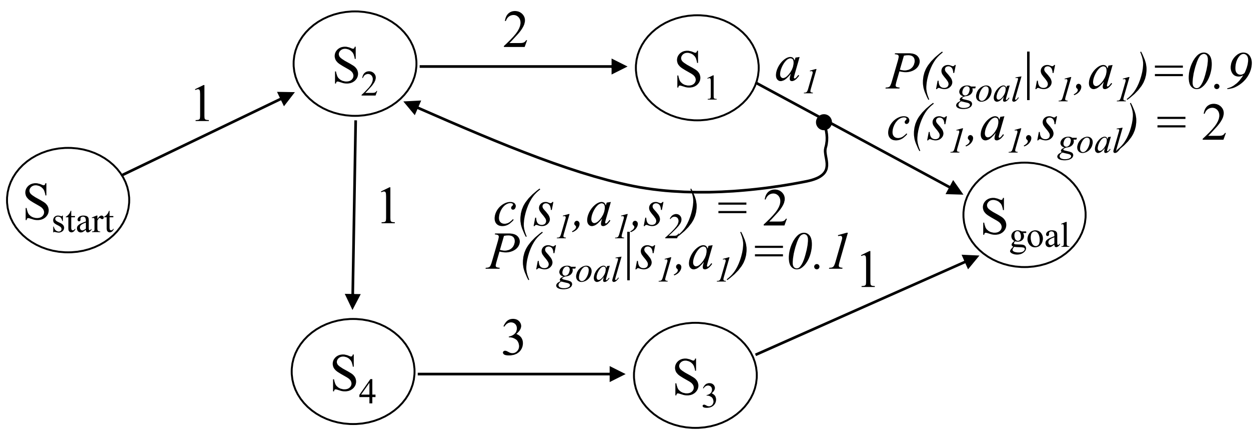


- Optimal policy $\pi^*$:

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  *Given a policy, its value can be computed by solving a system of linear equations*

  *expectation over outcome*

- expected cost-to-goal for $\pi_1$=(go through $s_4$

  *How to compute it?*

  1+1+3+1=6

- cost-to-goal for $\pi_2$=(try to go through $s_1$) is:

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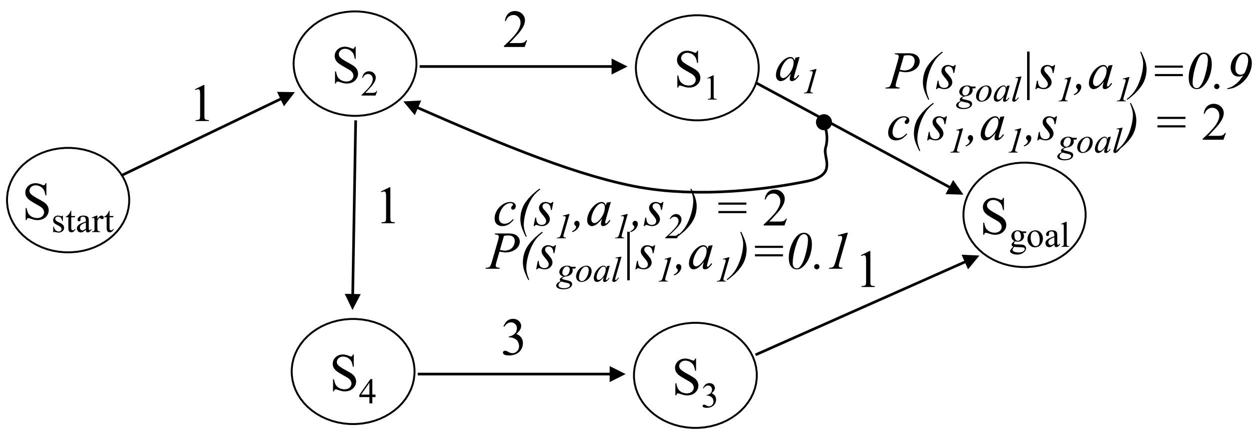


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  minimizes the *expected* cost-to-goal

*Given a policy, its value can be computed by solving a system of linear equations*

expectati...

*for $\pi_2$:*

$v(s_{start}) = 1+v(s_2)$

$v(s_2) = 2+v(s_1)$

$v(s_1) = 0.9*(2+v(s_{goal}))+0.1*(2+v(s_2))$

$v(s_{goal}) = 0$

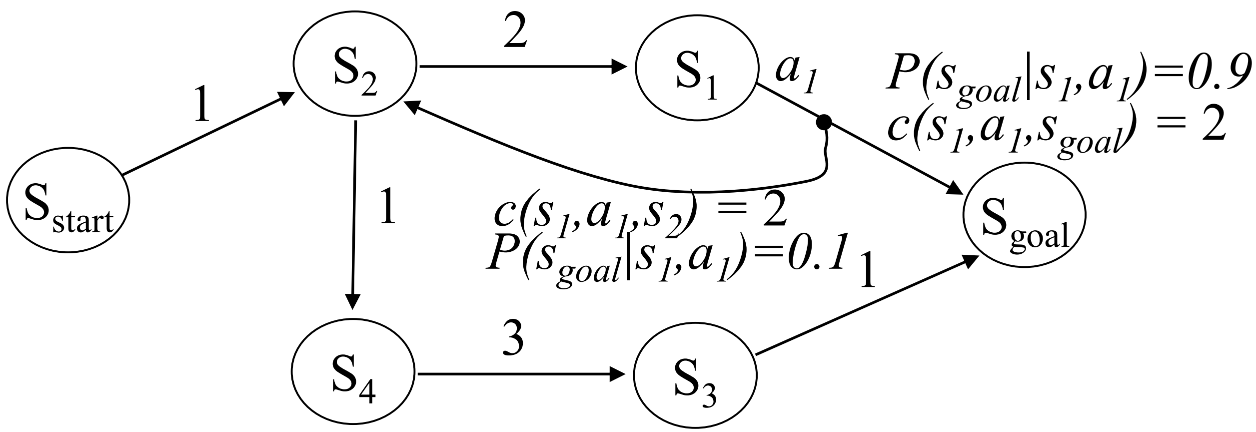
- expected cost-to-goal fo...

  1+1+3+1=6

- cost-to-goal for $\pi_2$=(try to g...

0.9*(1+2+2) + 0.9*0.1*(1+2+2+2+2) + 0.9*0.1 ... 2) + …=5.44$\overline{4}$

# Expected Cost Formulation

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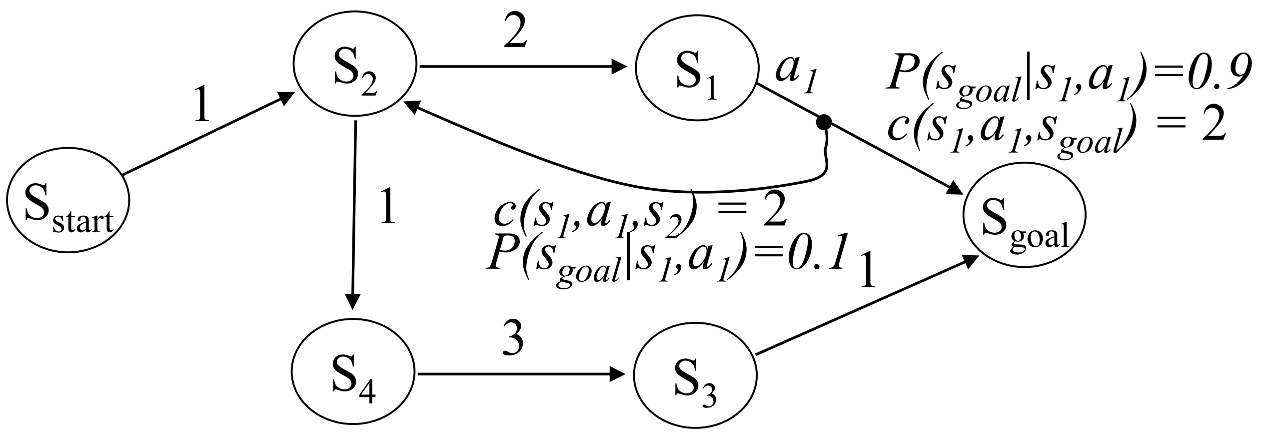


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*In contrast,*
*optimal policy for minimax formulation*
*was* $\pi_1$=(go through $s_4$)

# What You Should Know…

- Expected cost formulation for solving MDPs
- How to compute the expected cost of a given policy
- Expected cost vs. minimax cost formulations